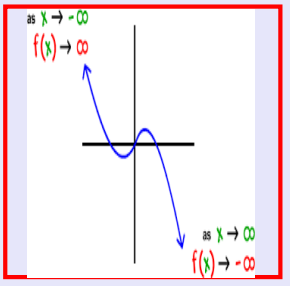
Math 245

Spring 2022

Lecture 24



Solving Quadratic Equations:

$ax^2 + bx + c = 0, \quad a \neq 0$

$x^2 + 8x + 15 = 0$

$2x^2 - 5x = 0$

$x^2 - 80 = 0$

Final Answer → Solution Set { , }

Square-Root Method:

If $x^2 = K$, then $x = \pm\sqrt{K}$

Solve $x^2 = 25$

$x = \pm\sqrt{25}$ $\quad \boxed{x = \pm 5} \Rightarrow \{\pm 5\}$

Solve $(2x-1)^2 = 49$

By S.R.M. $\Rightarrow$ $2x-1 = \pm\sqrt{49}$

$2x-1 = \pm 7$

$2x-1 = 7$ OR $2x-1 = -7$

$2x = 8$ $\quad 2x = -6$

$\boxed{x = 4}$ $\quad \boxed{x = -3}$

$\{-3, 4\}$

Solve $(3x+2)^2 = 50$

By S.R.M. $3x+2 = \pm\sqrt{50}$

$3x+2 = \pm\sqrt{25}\sqrt{2}$

$3x+2 = \pm 5\sqrt{2}$

$3x+2 = 5\sqrt{2}$ OR $3x+2 = -5\sqrt{2}$

$3x = -2+5\sqrt{2}$ $\quad 3x = -2-5\sqrt{2}$

$\boxed{x = \dfrac{-2+5\sqrt{2}}{3}}$ $\quad \boxed{x = \dfrac{-2-5\sqrt{2}}{3}}$

$\left\{\dfrac{-2 \pm 5\sqrt{2}}{3}\right\}$

Solve $x^2 + 36 = 0$

$x^2 = -36$

By S.R.M. $x = \pm\sqrt{-36}$

$x = \pm\sqrt{36}\sqrt{-1}$

$\boxed{x = \pm 6i}$ $\Rightarrow \{\pm 6i\}$

Solve $(5x-3)^2 + 10 = -6$

$(5x-3)^2 = -6 - 10 \Rightarrow (5x-3)^2 = -16$

Use S.R.M.

$5x - 3 = \pm\sqrt{-16}$

$\sqrt{-16} = \sqrt{16}\sqrt{-1} = 4i$

$5x - 3 = \pm 4i$

$5x = 3 \pm 4i$

$\boxed{x = \frac{3}{5} \pm \frac{4}{5}i} \Rightarrow \left\{\frac{3}{5} \pm \frac{4}{5}i\right\}$

Solve $(x+3)^2 + 65 = -10$

$(x+3)^2 = -10 - 65$

$(x+3)^2 = -75$

Use S.R.M. $x + 3 = \pm\sqrt{-75}$

$x = -3 \pm \sqrt{25}\sqrt{3}\sqrt{-1}$

$x = -3 \pm 5\sqrt{3}\,i$

$\boxed{x = -3 \pm 5i\sqrt{3}}$ $\Rightarrow \{-3 \pm 5i\sqrt{3}\}$

Making a perfect-square:

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

$a = 1$

$$x^2 + 8x + \left(\frac{8}{2}\right)^2 = \left(x + \frac{8}{2}\right)^2$$

$$x^2 + 8x + 16 = (x+4)^2$$

$$x^2 - 10x + \left(\frac{-10}{2}\right)^2 = \left(x + \frac{-10}{2}\right)^2$$

$$x^2 - 10x + 25 = (x-5)^2$$

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)^2$$

$\frac{5}{2}$

$$x^2 + 5x + \frac{25}{4} = \left(x + \frac{5}{2}\right)^2$$

$$x^2 - 7x + \left(\frac{-7}{2}\right)^2 = \left(x - \frac{7}{2}\right)^2$$

$$x^2 - 7x + \frac{49}{4} = \left(x - \frac{7}{2}\right)^2$$

$x^2 + \frac{2}{3}x + \left(\frac{1}{3}\right)^2 = \left(x + \frac{1}{3}\right)^2$

$\frac{1}{2} \cdot \frac{2}{3} = \frac{1}{3}$

$x^2 + \frac{2}{3}x + \frac{1}{9} = \left(x + \frac{1}{3}\right)^2$

$x^2 - \frac{3}{4}x + \left(\frac{-3}{8}\right)^2 = \left(x - \frac{3}{8}\right)^2$

$\frac{1}{2} \cdot \frac{-3}{4} = \frac{-3}{8}$ $\quad x^2 - \frac{3}{4}x + \frac{9}{64} = \left(x - \frac{3}{8}\right)^2$

Make a perfect-square:

1) $x^2 + 20x + 10^2 = (x + 10)^2$

$\frac{1}{2} \cdot 20 = 10$ $\quad x^2 + 20x + 100 = (x + 10)^2$

2) $x^2 - 9x + \left(\frac{9}{2}\right)^2 = \left(x - \frac{9}{2}\right)^2$

$\frac{1}{2} \cdot 9 = \frac{9}{2}$ $\quad x^2 - 9x + \frac{81}{4} = \left(x - \frac{9}{2}\right)^2$

3) $x^2 + \frac{3}{5}x + \left(\frac{3}{10}\right)^2 = \left(x + \frac{3}{10}\right)^2$

$\frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$ $\quad x^2 + \frac{3}{5}x + \frac{9}{100} = \left(x + \frac{3}{10}\right)^2$

Solve $x^2 + bx + c = 0$ by Completing the Square method:

$a = 1$

$x^2 + 6x + 13 = 0$

$x^2 + 6x + 3^2 = -13 + 3^2$ → $-13 + 9 = -4$

$(x + 3)^2 = -4$

Now use S.R.M.

$x + 3 = \pm\sqrt{-4}$

$x = -3 \pm 2i$

$\{-3 \pm 2i\}$

Solve $x^2 - 8x - 4 = 0$ by Completing the square method:

$x^2 - 8x - 4 = 0$

$\frac{1}{2} \cdot 8$

$x^2 - 8x + 4^2 = 4 + 4^2$

$(x - 4)^2 = 4 + 16$

$(x - 4)^2 = 20$

now use S.R.M.

$x - 4 = \pm\sqrt{20}$

$x = 4 \pm \sqrt{4}\sqrt{5}$

$x = 4 \pm 2\sqrt{5}$

$\{4 \pm 2\sqrt{5}\}$

Solve $x^2 - 3x - 28 = 0$ by Completing the Square method.

$x^2 - 3x - 28 = 0$

$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 28 + \left(\frac{3}{2}\right)^2$

$\frac{1}{2} \cdot 3 = \frac{3}{2}$

$28 + \frac{9}{4} = \frac{28 \cdot 4}{4} + \frac{9}{4} = \frac{112 + 9}{4} = \frac{121}{4}$

$\left(x - \frac{3}{2}\right)^2 = \frac{121}{4}$

Now use S.R.M.

$x - \frac{3}{2} = \pm\sqrt{\frac{121}{4}}$

$x = \frac{3}{2} \pm \frac{11}{2}$

$x = \frac{3+11}{2} = \frac{14}{2} = 7$

$x = \frac{3-11}{2} = \frac{-8}{2} = -4$

$\Rightarrow \{-4, 7\}$

Solve $x^2 - 6x + 10 = 0$ by Completing the Square method.

$x^2 - 6x + 3^2 = -10 + 3^2$

$\frac{1}{2} \cdot 6$

$(x - 3)^2 = -1$

Now use S.R.M.

$x - 3 = \pm\sqrt{-1}$

$\boxed{x = 3 \pm i} \Rightarrow \{3 \pm i\}$

We are working on SG 10